Equilibrium dynamics of the roughening transition in $^{(1+1)}$-dimensional systems

1998 J. Phys. A: Math. Gen. 31 2359

(http://iopscience.iop.org/0305-4470/31/10/011)

J. Phys. A: Math. Gen. **31** (1998) 2359–2369. Printed in the UK PII: S0305-4470(98)86957-6

Equilibrium dynamics of the roughening transition in (1 + 1)-dimensional systems

Yan-Chr Tsai

Institute of Physics, Academia Sinica, Taipei, Taiwan, Republic of China

Received 19 August 1997, in final form 20 November 1997

Abstract. The dynamics of the (1 + 1)-dimensional crystalline surface with long-range interactions is investigated using the renormalization group (RG) technique. The system in question displays a roughening transition which does not occur for systems with short-range interactions. The linear macromobility continuously decreases to zero as temperature decreases to the critical temperature in contrast to the usual two-dimensional roughening transition with a universal jump in the mobility at the transition point. The nonlinear mobility is also derived using the RG recursion relations. Two different RG schemes are employed and their differences are addressed.

1. Introduction

Recently intensive efforts have been made in the study of dynamics of growing interfaces. The particular attention paid to this problem stems not only from its technological importance and applications but also from its nontrivial temporal and spatial scaling behaviours. The processes of surface growth include particle deposition, sedimentation, epitaxial growth, solidification, etc. Due to the stochastic nature of growth processes, theoretical investigation of the surface growth can start from a variety of Langevin equations. Among those approaches nontrivial behaviour of surface growth have been explored [1]. The application of a numerical technique to this subject is intensive and the results are also fruitful [1].

The profile of a surface develops roughness gradually as stochastically deposited particles accumulate. To characterize the shape of a steady-state surface on a d-dimensional substrate, the two-point correlation function is usually introduced as:

$$S(x,t) = \langle [h(x+x_0, t+t_0) - h(x_0,t_0)]^2 \rangle \tag{1}$$

where $h(x,t)$ is the height of a surface with $x \in R^d$ and t is time. Scaling invariance of the equation of motion is manifested in the form of a two-point correlation function:

$$S(x,t) = x^{2\chi} f(t/x^z). \tag{2}$$

The roughness exponent χ describes the surface width in the long-time limit with the self-affine fractal structure and the dynamics exponent z characterizes the spread in time of disturbance on the surface. At the early stage of growth, the scaling function $f(y)$ becomes $By^{2\chi/z}$ with B being a constant, while at the saturation stage of growth, $f(y)$ changes into a constant A. The asymptotic scaling form of the correlation function is given by:

$$S(x, t \to 0) \to Ax^{2\chi} \tag{3}$$

$$S(x \to 0, t) \to Bt^{2\chi/z}. \tag{4}$$

0305-4470/98/102359+11$19.50

The height of the crystalline surface is not a continuous variable in the real world. The discrete nature of a lattice has been shown to play a significant role in both experiments and numerical simulations. A two-dimensional (2D) crystalline surface undergoes a so-called roughening transition at some temperature. Such transitions have been observed experimentally in plastic and metal crystals [2].

The theoretical explanation relies on the discrete Gaussian solid-on-solid (DGSOS) model [3]. The roughening transition is identified as a Kosterlitz and Thouless (KT) transition [3]. Two elements in this model are: (i) the surface tension stemming from the short-range interaction between atoms; and (ii) the periodic potential originating from the discreteness of a lattice. The nature of the phases of a crystalline surface is effectively determined by the dominance of one of those two terms. Below the transition temperature, the periodic potential is strongly relevant. The surface is pinned to the minimum of the potential and remains flat. Above the transition temperature, the periodic potential is irrelevant, and the surface is rough. The mobility of a growing surface at the transition point shows a so-called universal jump [3] in the KT transition.

The dimensional extension of the RG analysis of the 2D DGSOS model indicates that there still exists a roughening transition in one dimension [4]. However, this is just an artefact of dimensional extension. The one-dimensional (1D) DGSOS model is exactly solvable, and it has been shown that there is no phase transition. The latter approach is consistent with the result in statistical mechanics: one-dimensional systems with only short-range interactions should not have a phase transition [5]. Nevertheless, in nature there exists a variety of long-range interactions between atoms or molecules—for example the Van der Waals force, the effective force due to bulk strain, or long-range interactions in contact lines [2]. In general such interactions found to scale as a power law with the distances between atoms. The recent studies by Flament and Gallet (FG) [6] have shown that a possible long-range electrostatic interaction for a Langmuir crystalline film can lead to a roughening transition different from the usual 2D case.

The purpose of this paper is to investigate another possible case similar to the case FG: the crystalline-terraced surface with a special kind of long-range interactions between atoms. The main focus is to understand both the static and dynamical behaviours of a crystalline surface with long-range interactions displaying a roughening transition in one dimension where it is impossible for systems with short-range interactions.

This paper is organized as follows. In section 2 we motivate the study of a terraced surface with long-ranged interactions between steps. It is described by a 1D solid-on-solid (SOS) model including long-range interactions. In section 3 we establish a field-theoretic formulation of the nonlinear Langevin equation. The fluctuation and dissipation theorem and the derivation of the Martin *et al* (MSR) [7] action are touched upon. In section 4 we proceed with a detailed description of the renormalization procedures and the regularization scheme. Then we analyse the RG recursion relations of various physical parameters, and show that roughening transitions occur in one dimension. The behaviours of the mobility in both phases are also explored. In section 5 we adopt another RG scheme developed by Nozieres and Gallet (NG) [8], and obtain another version of the RG recursion relations. The differences between NG and the previous schemes are addressed. The implications of the RG recursion results for the physical quantities are discussed.

In the appendices we present the details of calculations. They include the basic analytical properties of the free correlation function, and the derivations of renormalization factors, Z_α and $\tilde{Z}$.

2. Long-range interaction model

As we mentioned in the introduction, the behaviour of the 2D DGSOS model, whose surface tension arises from the short-range interaction between atoms, is understood. Recently, effects of long-range interactions on roughening transitions have received much attentions [6, 9]. Here we study another possible case. In this section we introduce a 2D terraced surface model with parallel steps interacting with long-range force (the step meander configurations are neglected) and its effective Hamiltonian is reduced to that of a 1D interface.

To facilitate the discussion, we first consider the surface energy U of a crystalline surface as follows

$$U = \sum_{l,m,i,j} a_{lm}(n_{i+l,j+m} - n_{i,j})^2 \tag{5}$$

where $n_{i,j}$ is the height of the crystalline surface at the site (i, j). The form of a_{lm} determines the interactions between steps. For simplicity, we consider the surface with all steps parallel to the j-direction. One can derive the relation of the interaction $V(n)$ of two steps with the separation n and the variable a_{lm} [2] as follows

$$V''(n) = 2N_j \sum_m a_{nm} \tag{6}$$

where N_j is the site number along the j-direction of one step and $V''(n)$ is the second derivative of $V(n)$ with respect to n. The energy of the surface with steps indexed by $[10p]$ can also be written as [2]:

$$U = \sum_{lj} \tilde{a}_j(\tilde{n}_{l+j} - \tilde{n}_j)^2 \tag{7}$$

where $\tilde{a}_l = \frac{1}{2}V''(lp)$, and $\tilde{n}_j$ is the height fluctuation of step j from its position on the $[10p]$ surface. Equation (7) shows that the $[10p]$ surface steps can be depicted by a 1D DGSOS model.

Here we assume a special kind of long-range interaction which causes $\tilde{a}_l \sim \frac{1}{l^2}$. It is straightforward to generalize this model to other long-range interactions. By simply passing to the continuum limit equation (7) becomes [2, 10, 11]:

$$\mathcal{H}_0 = \frac{\rho}{2T}\int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}x' \left[\frac{h(x) - h(x')}{x - x'}\right]^2 = \frac{K}{2}\int_{-\infty}^{\infty} \frac{\mathrm{d}p}{2\pi}|p|h(p)h(-p) \tag{8}$$

where $\mathcal{H}_0$ is rewritten from U, $h(x)$ is the height of the line at position x, and ρ is the strength which characterizes the elastic energy due to long-range interactions between atoms. The temperature T has been absorbed into the Hamiltonian through $K = 2\pi\rho/T$.

A system described by $\mathcal{H}_0$ was studied in the context of quantum dissipations [10]. One of its recent applications is the electron system at a point contact of edge states in a quantum Hall liquid [12, 13].

For a terraced surface, the height h of every column of steps must be an integer multiple of the vertical unit spacing of the lattice. This discrete constraint leads to a periodic δ-function potential on h. The periodic potential here is taken as:

$$\mathcal{H}_\mathrm{p} = \frac{\alpha}{a}\int_{-\infty}^{\infty} \mathrm{d}x\ \cos[\gamma h(x)] \tag{9}$$

where α/a is the coefficient of the leading term (first harmonics) due to the discreteness, $\gamma = 2\pi/b$, and a and b are lattice constants parallel and perpendicular to the base line, respectively. After adding the periodic potential into the Hamiltonian, one can regard

the height of a surface as a continuous variable within the interval $(-\infty, \infty)$. As in the 2D DGSOS model, the higher harmonics are irrelevant near the critical point and can therefore be neglected in the long wavelength limit. The surface tension energy $\mathcal{H}_s = \frac{K'}{2} \int \mathrm{d}x\, [\nabla h(x)]^2$ is irrelevant and can be omitted beyond the lengthscale K'/K.

3. Equation of surface growth and MSR action

Recently it has been appreciated that the behaviours of surface growths can be revealed by variant Langevin equations. Most theoretical studies are based on this point of view. In the same spirit, we consider the dynamics of interface growth as governed by the following Langevin equation [3, 8, 14, 15]:

$$\frac{\partial h(x,t)}{\partial t} = \mu F - \mu \frac{\delta \mathcal{H}}{\delta h} + \eta(x,t) \tag{10}$$

where F is the driving force or the mean rate of deposition, μ is the mobility, and $\frac{\delta \mathcal{H}}{\delta h}$ describes the relaxation process of the surface after particle deposition. $\mathcal{H}$ is the sum of $\mathcal{H}_0$ and $\mathcal{H}_\mathrm{p}$. If $\mathcal{H}_\mathrm{p}$ is irrelevant, equation (10) is reduced to a linear stochastic equation, where the theory is exactly solvable with the roughness exponents $\chi = 0$ and the dynamical exponent $z = 1$ in one dimension. To mimic the stochastic nature of a surface growth process, η is introduced as the fluctuation of particle deposition from the mean rate. The noise is taken to be Gaussian distributed: $W[\eta] \propto \exp\{-\frac{1}{4\mu} \int \mathrm{d}x \int \mathrm{d}t\, [\eta(x,t)]^2\}$ with zero mean and short-ranged spatial and temporal correlations:

$$\langle \eta(x,t) \rangle = 0 \qquad \text{and} \qquad \langle \eta(x_1,t_1)\eta(x_2,t_2) \rangle = 2\mu\delta(x_1 - x_2)\delta(t_1 - t_2). \tag{11}$$

The coefficient 2μ is chosen such that the system will reach the equilibrium states weighted by the Boltzmann factor $\mathrm{e}^{-(\mathcal{H}_0 + \mathcal{H}_p)}$. Consequently the fluctuation–dissipation theorem (FDT) [16] is valid here in contrast to the case of far-from equilibrium surface growth [17], where equation (10) does not apply. (In general, one cannot find a proper Hamiltonian $\mathcal{H}$ for far-from equilibrium systems such that their equations of motion are deduced from equation (10).)

This stochastic equation will be analysed systematically by utilizing the MSR [7] formalism, where one introduces an auxiliary field $\tilde{h}$ to force equation (10) through a functional integral representation of a δ-function.

After averaging over $\eta(x,t)$, the generating functional for equation (10) takes the form:

$$Z[\tilde{J}, J] = \int \mathcal{D}\tilde{h}\mathcal{D}h \exp\{S_0[\tilde{h}, h] + S_\mathrm{int}[\tilde{h}, h] + \int \mathrm{d}x\, \mathrm{d}t\, (\tilde{J}\tilde{h} + Jh)\} \tag{12}$$

where the effective MSR action $\mathcal{S}_\mathrm{eff} (= \mathcal{S}_0 + \mathcal{S}_\mathrm{int})$ is given by:

$$\mathcal{S}_0 = \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t\, \mu\tilde{h}(p,t)\tilde{h}(-p,t) - \tilde{h}(-p,t) \left[\frac{\partial}{\partial t} h(p,t) - K\mu|p|h(p) \right] \tag{13}$$

$$\mathcal{S}_\mathrm{int} = -\frac{\mu\gamma\alpha}{a} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}t\, \tilde{h}(x,t) \sin[\gamma h(x,t)]. \tag{14}$$

When the above-mentioned δ-function is implemented, a Jacobian arises from a change of variables. However, this term can be cancelled by the contribution of diagrams containing a loop which is formed by a response propagator after the Wick contraction. Therefore the Jacobian can be neglected as long as one performs the calculation by excluding any diagrams with a loop from contracting a response propagator. One may calculate averaged correlation and response functions by differentiating the generating functional with respect to the current J or auxiliary current $\tilde{J}$ and setting $J = \tilde{J} = 0$ since $Z[J = \tilde{J} = 0] = 1$ [18, 19]. For simplicity, we will set the bare value of K to 1 for further calculations.

4. RG analysis of MSR action

In this section we follow the standard scheme developed by Amit *et al* [20] (AGG scheme). Before the RG analysis, it is beneficial to mention some useful analytic properties of propagators arising from the action.

The generating functional with only $e^{\mathcal{S}_0}$ is determined by two different types of free propagators—the free response function $R_0(x,t) = \langle h(x,t)\tilde{h}(0,0)\rangle$ and the free correlation function $C_0(x,t) = \langle h(x,t)h(0,0)\rangle$.

To regularize the infrared and ultraviolet divergence of the Feynman integral, we introduce mass term M for the field $h(x,t)$ and the short-distance cut-off a, respectively:

$$\begin{aligned} C_0(x,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{|p|+M}\mathrm{e}^{-\lambda(|p|+M)|t|}\mathrm{e}^{\mathrm{i}py}\Big|_{y^2=x^2+a^2} \\ &= -\frac{1}{\pi}[Ci(yM)\cos(yM)+Si(yM)\sin(yM)]+\frac{1}{\pi}\int_0^{\lambda|t|}\mathrm{d}u\,\mathrm{e}^{-Mu}\frac{u}{y^2+u^2} \end{aligned} \tag{15}$$

where $Ci(x) = -\int_x^\infty \frac{\cos t}{t}\,\mathrm{d}t$ and $Si(x) = -\int_x^\infty \frac{\sin t}{t}\,\mathrm{d}t$, and y is a positive number.

Next we should establish the relations of the bare and renormalized parameters in terms of so-called renormalization Z factors

$$\alpha = Z_\alpha \alpha_r \qquad h^2 = Z_h h_r^2 \qquad \tilde{h}^2 = Z_h^{-1}\tilde{Z}^2\tilde{h}_r^2 \tag{16}$$

$$\mu = Z_\mu Z_h \tilde{Z}^{-1}\mu_r \qquad M = Z_h^{-1}M_r. \tag{17}$$

By using the FDT [14, 15], we obtain $Z_\mu = 1$. The Z-factors are calculated in terms of the relations between the bare and renormalized vertex functions. The introduction of Z-factors is used to render the renormalized theory finite. Therefore the divergences of bare vertex functions will determine those in Z-factors. The detailed analysis is presented in appendices C and D. Following the conventional procedures [16, 21], we obtain:

$$Z_\alpha = 1 + \left(1 - \frac{\gamma^2}{2\pi}\right)\ln(\kappa a) + \mathrm{O}(\alpha^3) \tag{18}$$

$$Z_h = 1 \tag{19}$$

$$\tilde{Z} = 1 + \alpha_r^2\pi^2[\ln(\kappa a)]. \tag{20}$$

From the expansion of α the counter-terms here should be analytic in the momentum space. Therefore the long-range interaction in equation (8) does not suffer any renormalization, since this action is proportional to the absolute value of momentum. This fact holds for all orders of perturbative expansion of α. A momentum-shell approach due to Fisher and Zwerger [10] and a real-space calculation due to Kane and Fisher [12, 13] also yields the same result here.

The associated β functions are:

$$\beta_\mu = \kappa\left(\frac{\partial\mu}{\partial\kappa}\right)_b = \mu\kappa\left(\frac{\partial\ln\tilde{Z}}{\partial\kappa}\right)_b = \pi^2\alpha^2\mu \tag{21}$$

$$\beta_\alpha = \kappa\left(\frac{\partial\alpha}{\partial\kappa}\right)_b = -\alpha\kappa\left(\frac{\partial\ln Z_\alpha}{\partial\kappa}\right)_b = \alpha\left(\frac{\gamma^2}{2\pi}-1\right). \tag{22}$$

Using the rescaling factors $x \to \mathrm{e}^l x$ and $t \to t\mathrm{e}^{lz}$ ($l = \ln y$ where y is the rescaling factor), the RG recursion relations of various parameters are given by:

$$\frac{\mathrm{d}K}{\mathrm{d}l} = 0 \tag{23}$$

$$\frac{\mathrm{d}F}{\mathrm{d}l} = F \tag{24}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}l} = \left(1 - \frac{\gamma^2}{2\pi}\right)\alpha + \mathrm{O}(\alpha^3) \tag{25}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}l} = (z - 1 - (\pi\alpha)^2)\mu. \tag{26}$$

The flow diagram of the parameter α has been obtained by Fisher and Zwerger [10]. Based on the duality property of the density matrix, the flows of α go to infinity above the transition point while the dual counterparts (those in the regions below the transition) are driven to zero [10]. The higher-order calculations for α can be avoided. The flows of α above the transition point are implied by equation (25).

The analysis of the RG recursion relations is obvious here. First one integrates out the recursion equations. Above the transition, as explained above, the coupling constant α flows to infinity. On a sufficiently large lengthscale, a mass term generated by the relevant cosine term will dominate over the surface energy strength K. The interface is pinned at the minima of the periodic potential, and remains in a flat phase as in $(2+1)$-dimensional systems. Although there is no infrared fixed point of α for $\tau = 1 - \frac{\gamma^2}{2\pi} > 0$, the relation $\alpha(l) = \alpha(0)\mathrm{e}^{\tau l}$ where $\alpha(0)$ is the bare value of the parameter α should hold as long as $l \ll 1/\tau = l_D$.

Next we turn to the dynamical properties of the interface in both phases. To facilitate the discussion, we obtain the expression of μ on the lengthscale l by integrating equations (25) and (26):

$$\mu(l) = \mu(0)\mathrm{e}^{-\frac{\pi^2\alpha(o)^2}{2\tau}(\mathrm{e}^{2\tau l}-1)}. \tag{27}$$

The linear response mobility (as $F \to 0$) can be obtained by letting $l \to \infty$ in equation (27). We find:

$$\mu_M = \mu(0)\mathrm{e}^{\frac{\pi^2\alpha(0)^2}{2\tau}} \qquad \text{for } \tau < 0 \tag{28}$$

and

$$\mu_M = 0 \qquad \text{for } \tau > 0. \tag{29}$$

Here $\mu(0)$ represents the bare value of the parameter μ. In contrast to the roughening transition in $2+1$ dimensions, the linear response mobility decreases to zero as temperature approaches the transition point from above.

Therefore there is no so-called universal jump of the mobility for the 1D case. This particular behaviour compared with the 2D case is due to the non-renormalization of surface energy strength K. The higher-order correction will not change this behaviour qualitatively. In the case where the applied force F is finite, the movement of an interface is characterized by the nonlinear response mobility. Since F is a relevant field, on some sufficiently large lengthscale $l^* \sim -\ln F$, it will grow significantly. The pinning effect due to the periodic potential will be wiped out by the motion of an interface. Beyond this scale, the effect of the strength α, on the average, can be neglected. The mobility μ will scale normally, and its canonical dimension is zero. Thus, the nonlinear response mobility is yielded by stopping the RG iteration at this scale:

$$\mu(l^* \sim -\ln F) \sim \mu(0)\exp\left[-\frac{C}{\tau F^{2\tau}}\right] \qquad \text{when } \tau > 0 \tag{30}$$

where C is a constant whose value depends on the bare values of other parameters. Meanwhile l^* should be less than l_D, i.e. $-\ln F < 1/\tau$. This kind of nonlinear response

has been of interest in the study of I–V characteristics of the vortex glass phase of type II superconductors [22]. Here we see that it can arise even in the pure systems with long-range interactions. Since in both systems the static Gaussian action does not suffer the renormalization, they share the common forms of nonlinear response.

In the region far below the transition point, the results presented here in general do not hold. The role of higher harmonics neglected at the beginning of the RG analysis is important in this region. However, one can resort to the activated-dynamics mechanism for systems far below the transition temperature.

5. Nozieres and Gallet scheme

In this section we adopt another RG scheme employed by Nozieres and Gallet (NG) to investigate the 2D sine-Gordon model. The NG scheme does not require expansion around the transition point ($\delta = 0$), in contrast to the AGG scheme [20]. However, the NG scheme adopted the momentum shell scenario, which is notorious for high-order perturbation calculations.

Let us start with equation (10). In general, the surface height $h(x,t)$ is a functional of the given noise $\eta(x,t)$ determined by the equation of motion. The dependence of the height h on the noise η is highly nonlinear. It is quite natural to employ the RG method to handle the fluctuations of nonlinearity.

To perform the RG calculations, one first divides the noise η into low- and high-momentum components:

$$\eta = \bar{\eta}(k) + \Delta\eta(k) \tag{31}$$

where $\bar{\eta}$ associates with the low-momentum part with $|k| < \Lambda$ and $\Delta\eta$ with the high-momentum part with $\bar{\Lambda} < |k| < \Lambda$. $\bar{\Lambda}$ is defined as: $\bar{\Lambda} = (1-\epsilon)\Lambda$, where ϵ is an infinitesimal small parameter same as dl in the denominator of equation (23). Here we use a sharp cut-off Λ in the momentum space.

One then integrates out the low-momentum part $\Delta\eta(k)$, and η comes out with:

$$\bar{h} = \langle h[\bar{\eta} + \Delta\eta]\rangle_{\Delta\eta} \tag{32}$$

where the angle bracket represents the average over the high-momentum part. To facilitate the calculation, we also denote $\Delta h = h - \bar{h}$. The average of equation (10) over the high-momentum noise yields:

$$\frac{\partial}{\partial t}\bar{h}(p,t) = -\mu\frac{\delta\mathcal{H}_0}{\delta h} + \bar{\eta} + \langle\Upsilon\rangle_{\Delta\eta} \tag{33}$$

where $\Upsilon = \mu\frac{\partial\mathcal{H}_p}{\partial h}$. Subtracting equation (10) from equation (33), one obtains:

$$\frac{\partial}{\partial t}\Delta h(x,t) = -\mu\frac{\delta\mathcal{H}_0[\Delta h]}{\delta(\Delta h)} + \Delta\eta + \Upsilon - \langle\Upsilon\rangle_{\Delta\eta}. \tag{34}$$

Since the momentum shell is infinitesimally small here ($\bar{\Lambda} \to \Lambda$), Δh is also infinitesimally small, and we can expand $\langle\Upsilon\rangle$ as:

$$\langle\Upsilon\rangle_{\Delta\eta} = -\gamma\mu\alpha\sin(\gamma\bar{h})\left[1 - \frac{\gamma^2}{2}\langle\Delta h^2\rangle + \cdots\right] \tag{35}$$

and

$$\Delta\Upsilon = \Upsilon - \langle\Upsilon\rangle_{\Delta\eta} = -\gamma^2\mu\alpha\cos(\gamma\bar{h})\Delta h + \cdots. \tag{36}$$

Solving equation (34), one inherits the expression for Δh:

$$\Delta h(x,t) = \int_{-\infty}^{\infty} \mathrm{d}x' \int_{-\infty}^{t} \mathrm{d}t'\, R_0(x-x', t-t')[\Delta\Upsilon(x',t') + \Delta\eta(x',t')] \tag{37}$$

where $R_0(x-x', t-t') = \frac{1}{\pi}[\frac{\mu|t-t'|}{(\mu|t-t'|)^2+(x-x')^2}]$. By expanding $\Delta h(x,t)$ and $\Delta\Upsilon$ in orders of α, one has the leading order of Δh contributed only from $\Delta\eta$, since Υ is at least of the order of α as shown in equation (36). To obtain the renormalization of K and the mobility μ from the periodic potential strength, one should evaluate $\langle\Upsilon\rangle$ in equation (33).

Up to order α^2, we have $\langle\Delta h^2\rangle = 2\langle\Delta h^{(0)}\Delta h^{(1)}\rangle$. By employing the expression $\Delta h^{(1)}(x,t) = \int_{-\infty}^{\infty}\mathrm{d}x'\int_{\infty}^{t}\mathrm{d}t'\, R_0(x-x', t-t')[-\gamma^2\frac{\mu\alpha}{a}\cos(\gamma\bar{h})\Delta h^{(0)}]$ and equation (35), we obtain:

$$\langle\Upsilon^{(2)}(x,t)\rangle = -\frac{\gamma^5\mu^2\alpha^2}{a^2}\int_{-\infty}^{\infty}\mathrm{d}x'\int_{-\infty}^{t}\mathrm{d}t'\, R_0(x-x', t-t')\cos[\gamma\bar{h}(x',t')] \times \sin[\gamma\bar{h}(x,t)]\langle\Delta h^{(0)}(x',t')\Delta h^{(0)}(x,t)\rangle \tag{38}$$

where $\langle\Delta h^{(0)}(x',t')\Delta h^{(0)}(x,t)\rangle = \frac{\epsilon}{\pi}\cos(\Lambda|x-x'|)\mathrm{e}^{-\mu\Lambda|t-t'|}$. The product of sine and cosine terms can be decomposed as the sum of $\sin\{\gamma[\bar{h}(x,t)+\bar{h}(x',t')]\}$ and $\sin\{\gamma[\bar{h}(x,t)-\bar{h}(x',t')]\}$. The former term can be neglected since it contains higher harmonics and is therefore irrelevant on the long wavelength scale. The latter term will renormalize the mobility μ seen from the expansion of equation (38):

$$\gamma\left[\frac{\rho^2}{4}\nabla^2\bar{h} + \tau\frac{\partial\bar{h}}{\partial t}\right]\langle\cos\{\gamma[(\bar{h}(x',t')-\bar{h}(x,t)]\}\rangle \tag{39}$$

where the gradient term will not renormalize the elastic term in equation (8), which is proportional to the absolute value of momentum. The non-renormalization of K is also justified in the present scheme. The cosine term in equation (39) is due to higher-order corrections. This cosine term will result in $\mathrm{e}^{-\gamma^2 S(\rho,\tau)}$, where $S(\rho,\tau) = \langle[\bar{h}(x+\rho,t+\tau)-\bar{h}(x,t)]^2\rangle = \frac{2}{\pi}\int_0^{\Lambda}\frac{\mathrm{d}k}{k}[1-\cos(k\Lambda)\mathrm{e}^{-\mu k\tau}]$.

The recursion relation of the mobility will be

$$\frac{\mathrm{d}\mu}{\mathrm{d}l} = \frac{2\gamma^6}{\pi^2 a^2}BV^2\mu \tag{40}$$

where $V = \frac{\alpha}{\Lambda}$, $B = \int_0^{\infty}\int_0^{\infty}\mathrm{d}x\,\mathrm{d}y\,\frac{xy^2}{y^2+1}\cos(x)\mathrm{e}^{-\gamma^2 S(x,y)}\mathrm{e}^{-xy}$ with the change of variables $x = \Lambda\rho$, $y = \mu\tau/\rho$, and $S(x,y) = \frac{2}{\pi}\int_0^1\frac{\mathrm{d}k}{k}(1-\cos(kx)\mathrm{e}^{-xy})$.

The prediction of the critical dynamics behaviours can be obtained by following the same approach that we pursued in section 4. As shown in equation (40), the coefficient preceding V^2 is different from that in equation (26), indicative of the non-universality of mobility expression. Two schemes give different coefficients, as occurs in the 2D sine-Gordon theory [8, 15]. Nevertheless, the critical behaviours predicted by two schemes are qualitatively the same. NG used the momentum shell of the noise, while AGG integrated out the noise to obtain the generating functional and imposed cut-offs on the field variables.

In the AGG scheme the theory is expanded around the transition point, while in the NG scheme this expansion is not necessary. The latter is supposed to still hold in the region far away (but not very far) from the transition temperature. The critical behaviour predicted by the two schemes are qualitatively consistent. The difference in physical properties of parameters predicted from two schemes can be seen by integrating the corresponding recursion relations.

However, the AGG scheme provides an obviously consistent field-theoretic approach, which will require less effort to extend the calculation to higher order in α.

6. Conclusion

We summarize the main conclusion of our investigation of the equilibrium dynamics of a 1D crystalline surface with a long-range interaction. There exists a phase transition between a rough phase at high temperature, and a flat phase at low temperature. In the rough phase, the correlations and response function possess the same scaling behaviour as in the free case (with the periodic potential). The periodic potential is irrelevant in this phase. As the transition temperature T_c approaches from above, the macroscopic mobility vanishes continuously (in contrast to the 2D roughening transition where it has a finite jump). This particular behaviour is due to the long-range interaction, which not only leads the system to a roughening transition in one dimension and but also forbids the renormalization of K from a periodic potential. This implies that the surface is stiffer than that with short-range interactions.

The properties of the low-temperature phase are qualitatively similar to those found in 2D roughening transitions. The height–height correlations have a short-range correlation length e^{l_D}. In other words, a mass term is generated in this strong coupling region. Near the transition point, the linear mobility characterized by the ratio of the driving force and the velocity of a moving surface vanishes.

We also adopt the NG scheme to calculate the RG recursion relations. In both AGG and NG schemes one obtains the same roughening temperature. The AGG approach can provide more accurate predictions than the AGG approach in the lower-temperature region far below the transition. However, in the case with larger coupling constants α the AGG scheme provides easier extensions to higher-order perturbative calculations thus one can obtain more accurate results near the transition temperature.

Acknowledgments

The author is grateful to T Hwa for intense discussion and insightful suggestions. He would also like to thank C-N Chen and Y Shapir for useful discussions. The work was partially supported by the National Science Council of Taiwan under grant nos NSC 84-2112-M-001-013 Y and NSC 84-2112-M-001-048.

Appendix A. Analytic properties of $C_0(x, t)$

We have defined the regularization for $C_0(x, t)$ in equation (15). A further simplification can be obtained by reparametrization.

$$\begin{aligned} C_0(x,t) &= \frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{|p|+M}\mathrm{e}^{-\mu(|p|+M)|t|}\mathrm{e}^{\mathrm{i}py}\Big|_{y^2=x^2+a^2} \\ &= \frac{2}{2\pi}\int_0^{\infty}\frac{\mathrm{d}p}{p+M}\mathrm{e}^{-\mu(p+M)|t|}\cos(\mathrm{i}py) \\ &= \frac{1}{\pi}\int_0^{\infty}\int_0^{\infty}\mathrm{d}p\,\mathrm{d}\alpha\,\mathrm{e}^{-\alpha(p+M)}\mathrm{e}^{-\mu(p+M)|t|}\cos(\mathrm{i}py) \\ &= \frac{1}{\pi}\int_{\mu|t|}^{\infty}\mathrm{d}u\,\mathrm{e}^{-Mu}\frac{u}{u^2+y^2} \\ &= \frac{1}{\pi}\int_0^{\infty}\mathrm{d}u\,\mathrm{e}^{-Mu}\frac{u}{u^2+y^2} - \frac{1}{\pi}\int_{\mu|t|}^{0}\mathrm{d}u\,\mathrm{e}^{-Mu}\frac{u}{u^2+y^2} \end{aligned} \tag{A1}$$

where the first term of the last line in equation (A1), can be written as: $-\frac{1}{\pi}[Ci(yM)\cos(yM) + Si(yM)\sin(yM)]$. Around $x = 0$, $Ci(x) = C + \log(x) + \sum_{k=1}^{\infty}(-1)^k \frac{x^{2k}}{2k(2k!)}$, and $Si(x) = -\frac{\pi}{2} + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^{2k-1}}{(2k-1)(2k-1!)}$. Here C is the Euler's constant.

In the limit of $M\mu t \ll 1$, the second term of the last line in equation (A1) can be approximated by:

$$-\frac{1}{\pi}\int_0^{\mu|t|} \mathrm{d}u\, \mathrm{e}^{-Mu}\frac{u}{u^2+y^2} \tag{A2}$$

$$\sim -\frac{1}{\pi}\int_0^{\mu|t|} \mathrm{d}u\, \frac{u}{u^2+y^2} \tag{A3}$$

$$= -\frac{1}{2\pi}\log\left[\frac{(\mu|t|)^2+y^2}{y^2}\right] + \mathrm{O}(\mu t). \tag{A4}$$

Appendix B. The vertex functions

The bare and renormalized vertex functions [16, 21] can be related by factors of Z_h, $Z_{\tilde{h}}$. For instance,

$$\Gamma^r_{N,L}(q,\omega;\nu_r,M_r,\kappa) = (Z_h)^{\frac{L}{2}}(Z_{\tilde{h}})^{\frac{N}{2}}\Gamma_{N,L}(q,\omega;\nu_0,M_0,a) \tag{B1}$$

where ν_r and ν_0 label renormalized parameters ($\alpha_r, \mu, \ldots$) and bare parameters ($\alpha, \mu_0, \ldots$), respectively. q and ω are the external momentum and frequency, respectively. In the corresponding vertex function, a is a short-distance cut-off, and κ is a mass scale. $\Gamma_{N,L}$ stands for the vertex function with L external h lines and N external $\tilde{h}$ lines. The factors, Z_h and $Z_{\tilde{h}}$, are set to remove the divergent parts of the vertex function Γ.

The generating functional of the vertex functions can be expressed in orders of the perturbation parameters. The expressions are referred to elsewhere [14, 15, 23].

Appendix C. The calculation of Z_α

The generating functional of vertex functions $P[M,\tilde{M}]$ is expanded as: $P[M,\tilde{M}] = \langle V[h+M,\tilde{h}+\tilde{M}]\rangle + \mathrm{O}(\alpha^2)$, where $V[h,\tilde{h}] = -\frac{\gamma\mu\alpha}{a}\iint \mathrm{d}x\,\mathrm{d}t\,\tilde{h}(x,t)\sin\gamma h(x,t)$.

The Wick contraction of $V[h,\tilde{h}]$ in the leading order is: $\langle V[h+M,\tilde{h}+\tilde{M}]\rangle = V[M,\tilde{M}]\langle\cos[\gamma h(x,t) - \gamma h(x',t']\rangle = VJa$, where

$$J = \frac{1}{a}\exp[-C_0(x=0,t=0)] = (sM)(sMa)^{-(\frac{\gamma^2}{\pi}-1)} \tag{C1}$$

with $s = \mathrm{e}^C$ and we have used the singular behaviour of $C_0(0,0)$ as implied by equation (A1).

With $g = Z_g g_r$ we have $Z_g = (\kappa a)^{1-\frac{\gamma^2}{\pi}}$.

Appendix D. The calculation of vertex function $\Gamma_{1,1}(q,\omega)$

The expression for the vertex function $\Gamma_{1,1}(q,w)$ can be obtained in the same way as in the 2D case [14, 15]. We expand it up to the second order of α,

$$\Gamma_{1,1}(q,\omega) = -\mathrm{i}\omega + \mu(|q|+M) + \alpha\mu J + \mu(\alpha J)^2\left[\int_{-\infty}^{\infty}\mathrm{d}x\,\{P(x,0) - \mathrm{e}^{-\mathrm{i}qx}Q(x,0)\right.$$

$$-\mathrm{i}\omega \int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}(qx-\omega t)} Q(x,t)\Bigg] \tag{D1}$$

where $P(x,t) = \cosh\{\gamma^2 C_0(x,t)\} - 1$, and $Q(x,t) = \sinh\{\gamma^2 C_0(x,t)\} - \gamma^2 C_0(x,t)$.

To obtain the information of the dynamics, we only need to extract the divergence of the term containing $Q(x,t)$ by using $\Gamma^r_{1,1} = \tilde{Z}\Gamma_{1,1}$.

For the time being, we neglect the prefactor. The singular term from the last term in equation (D1) can be obtained as:

$$\begin{aligned}
-\mathrm{i}\omega \int_0^\infty \mathrm{d}t \int_{-\infty}^\infty \mathrm{d}x\, \frac{1}{2}\left[\mathrm{e}^{-\frac{\gamma^2}{2\pi}\log[s^2(x^2+a^2)M^2]}\mathrm{e}^{-\frac{\gamma^2}{2\pi}\log(\frac{\mu^2t^2+y^2}{y^2})}\right] \\
&= -\mathrm{i}\omega \int_0^\infty \mathrm{d}t \int_{-\infty}^\infty \mathrm{d}x\, \frac{1}{2}\left[\frac{1}{M^2s^2(x^2+a^2)}\mathrm{e}^{-\log(\frac{\mu^2t^2+y^2}{y^2})}\right] \\
&= -\mathrm{i}\omega \int_0^\infty \mathrm{d}t \int_{-\infty}^\infty \mathrm{d}x\, \frac{1}{2}\left[\frac{1}{M^2s^2(x^2+a^2)}\frac{y^2}{y^2+\mu^2t^2}\right] \\
&\sim -\mathrm{i}\omega \int_0^\infty \frac{\mathrm{d}\tilde{s}}{\mu}\sqrt{x^2+a^2}\frac{1}{2}\int_{-(sM)^{-1}}^{(sM)^{-1}} \mathrm{d}x\, \frac{1}{(sM)^2\sqrt{x^2+a^2}}\frac{1}{\tilde{s}^2+1} \\
&= -\mathrm{i}\omega\frac{1}{2\mu}\int_0^\infty \frac{\mathrm{d}\tilde{s}}{\tilde{s}^2+1}\left[2\int_0^{(sM)^{-1}} \mathrm{d}y\, \frac{1}{(sM)^2}\frac{1}{\sqrt{x^2+a^2}}\right] \\
&= -\mathrm{i}\omega\frac{1}{2\mu}\int_0^\infty \frac{\mathrm{d}\tilde{s}}{\tilde{s}^2+1}2\times\left[-\frac{1}{(sM)^2}\log(sMa)\right]
\end{aligned}$$

where we have substituted $\tilde{s} = \frac{\mu t}{\sqrt{x^2+a^2}}$.

With the prefactor, $\tilde{Z} = 1 + 2\pi\alpha_r^2 b \ln(\kappa a)$, where $b = \int_0^\infty \mathrm{d}\tilde{s}\, \frac{1}{\tilde{s}^2+1} = \frac{\pi}{2}$.

References

[1] Family F and Vicsek T 1991 *Dynamics of Fractal Surfaces* (Singapore: World Scientific)
[2] Schulz H J 1985 *J. Physique* **46** 275
[3] Chui S T and Weeks J D 1978 *Phys. Rev. Lett.* **48** 733
Mazzeo G, Carlon E and Vanbeijeren H 1996 *Surf. Sci.* **352** 960
[4] Grinstein G and Lee D-H 1991 *Phys. Rev. Lett.* **66** 177
[5] Lenard A 1961 *J. Math. Phys.* **2** 682
[6] Flament C and Gallet F 1992 *Europhys. Lett.* **20** 331
[7] Martin P C, Siggia E and Rose H 1973 *Phys. Rev.* A **8** 423
[8] Nozieres P and Gallet F 1987 *J. Physique* **48** 353
[9] Kolomeisky E B and Curcic T 1995 *Phys. Rev. Lett.* **75** 1775
[10] Fisher M P A and Zwerger W 1985 *Phys. Rev.* B **32** 6190
[11] Tsai Y C 1995 *Phys. Rev.* E **51** R2723
[12] Kane C L and Fisher M P A 1992 *Phys. Rev. Lett.* **68** 1220
[13] Kane C L and Fisher M P A 1992 *Phys. Rev.* B **46** 15 233
[14] Neudecker B 1982 *Z. Phys.* B **49** 57
[15] Neudecker B 1982 *Z. Phys.* B **48** 149
[16] Zinn-Justin J 1989 *Quantum Field Theory and Critical Phenomena* (New York: Oxford Science)
[17] Kardar M, Parisi G and Zhang Y C 1986 *Phys. Rev. Lett.* **56** 898
[18] Janssen H K 1976 *Z. Phys.* B **23** 377
[19] Bausch B, Jassen H K and Wanger H 1976 *Z. Phys.* B **24** 113
[20] Amit D J, Goldschmidt Y Y and Grinstein G J 1980 *J. Phys. A: Math. Gen.* **13** 585
[21] Amit D J 1984 *Field Theory, the Renormalization Group, and Critical Phenomena* (Singapore: World Scientific)
[22] Tsai Y C and Shapir Y 1992 *Phys. Rev. Lett.* **69** 1773
[23] Tsai Y C and Shapir Y 1994 *Phys. Rev.* E **50** 3546